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EXPERIMENTAL STUDY OF VARIATIONS IN THE THICKNESS OF A LIQUID FILM MOVING OVER THE INNER SURFACE OF A ROTATING CYLINDER

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The behavior of a liquid layer moving in a mass-force field on the inner surface of a rotating vertical cylinder is studied experimentally. Free-surface profiles of the liquid moving under these conditions are constructed. An empirical dependence for the mean thickness of the film is obtained in criterial form. The presence of a hydraulic jump in the lower part of the cylinder behind the entrance of the liquid onto the vertical surface is revealed.

Many technological devices used to intensify heat and mass transfer in a liquid or to spray it [1–8] are based on the phenomenon of motion of thin films in a field of centrifugal forces. In particular, when a glass-forming material is melted in a rotated plasmachemical reactor for producing mineral fiber, the molten mass moves over the inner surface of the device [8]. The properties of the material of the fibers produced are formed directly in the film moving under thermal and dynamic loading. Knowledge of the liquid-film thickness and its distribution along the axial coordinate in motion over the inner surface of a rotating cylinder allows one to calculate the heat exchange of such a film device.

For an inner rotating cylindrical surface, Gol'dshtik [1] developed a theory of centrifugal sprayers, based on a model of potential motion of an inviscid liquid. However, comparison of the theoretical dependences obtained was performed with experimental results of studies of liquid-film spreading over a horizontal surface under gravity only. For viscous flows over an inner rotating cylindrical surface, there have been few experimental studies, and they poorly agree with theoretical studies. Leppert and Nimmo [3] calculated heat transfer for condensation of vapor on a liquid film moving down a horizontal plate. It is shown that even when the mass force is normal to the condensation surface, heat removal can have a finite value if running-off on the edges is allowed. In this case, the thickness of the condensate film and the resultant heat removal are controlled by varying the hydrostatic pressure across the film thickness. These results are applicable to condensation inside a rotating cylinder provided that the surface curvature is small and the rotational speed is high. However, to close a solution that describes this model, it is necessary to invoke two empirical parameters: the initial and final thicknesses of the condensate film. Postnikov [4] obtained a solution for the thickness of a laminar film moving over the inner surface of a rotating cylinder that requires one experimental value — the initial thickness of the film. Both solutions [3, 4] have one feature in common: a sharp decrease to zero in the thickness of the moving liquid film, which is typical of a central flow region on the longitudinal coordinate.

Early experimental studies in this field [5, 6] were concerned with liquid flow in a rotating tube with broad rings at the edges. Dependences of the device capacity on the flow rate and angular frequency of rotation were obtained. Flow visualization using tracers in the form of a tinted liquid showed that the flow is of two-layer character. The main mass flow proceeded in a thin layer located closer to the center from the edge of the rings. The effect of viscosity on liquid flow in a rotating cylinder with various methods of feeding the liquid was studied experimentally in [7]. The results are somewhat unexpected: the thickness of the moving layer decreases with increase in viscosity. This effect was explained by "slipping" of the moving liquid layer

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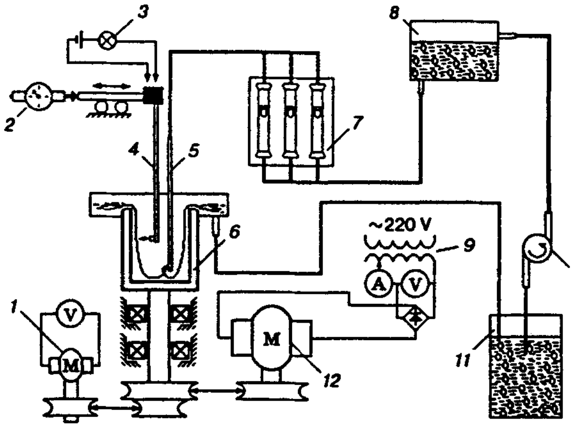

Fig. 1. Diagram of experimental setup: 1) tachometer generator; 2) indicator; 3) light-emitting diode; 4) rod; 5) branch pipe; 6) cylinder; 7) rotameters; 8) tank; 9) autotransformer; 10) pump; 11) reservoir; 12) direct current engine.

under two-layer flow conditions. In the present paper, we give a different interpretation of this effect, based on experimental data.

An experimental facility (Fig. 1) was designed to study liquid-film flows with constant viscosity over the inner surface of a rotating cylinder. The working element of the facility is hollow rotating cylinder 6. For installation of changeable cylinders, the facility is provided with a special mounting seat, which is fastened to the centering cone of a shaft. All critical parts, including the working cylinders, were of the 8th accuracy class, and the mating parts were fitted to them to ensure coaxiality to within 0.01 mm. When the cylinders were put in the operative position, coaxiality was additionally controlled by means of a dial indicator, which was fastened to the facility, and balancing was performed by adjusting screws. Three cylinders of height 0.19, 0.23, and 0.25 m and radius 0.04, 0.0515, and 0.0375 m, respectively, were used.

Distilled water at room temperature was used as the model liquid because of its ready availability and well-studied physical properties. The temperature of the liquid was constantly monitored by a laboratory thermometer with an accuracy of 0.1°C. In processing of experimental data, the temperature variations in the course of the experiments were taken into account by means of special corrections in the form of a polynomial dependence of the water viscosity on the temperature measured in the experiment. The rotational speed of the cylinder was smoothly changed by varying the speed of the direct-current engine 12 using autotransformer 9. Control and stabilization of the rotation frequency of the cylinder were performed by means of electromechanical tachometer generator 1, which governed a stabilizing device connected to the supply circuit of the direct-current electric engine. Since the tachometer generators have a rather high error referred to the upper limit of measurements (up to 4%), the system for controlling the rotation frequency was provided with a switch of channels, which divided the entire range of the output signal into approximately equal intervals. Each interval was subjected to separate calibration, which reduced the relative methodical error in the measurements of the rotational speed of the cylinder to 1.5%.

The model liquid from tank 8 was carried through rotameter unit 7 by branch pipe 5 at the bottom of the rotating volume. A thin layer of the liquid moved in the interior cavity and was sprayed from the upper edge to the receiving device. Return supply of the liquid from reservoir 11 was accomplished by pump 10. The flow rate was measured by PC IV type rotameters, whose relative error amounts to 4%. Additional floats made of materials with different densities were inserted into the standard rotameters to decrease the measurement error. The readings of each of the floats were calibrated separately, and this decreased the total relative error to 1.5%.

The local film thickness on the axial coordinate was measured by the contact method, which was somewhat upgraded. The location of the free surface of the flow was recorded when the contact between a silver disk and a thin elastic wire deflected by the rotating liquid layer closed. The length and elasticity of the wire were selected experimentally on a transparent cylinder. The criterion of the moment of contact was the appearance of the first perturbation waves around the end of the rod. The following optimum characteristics of the sensor were established: wire length 10–12 mm and its diameter 0.07 mm.

The sensor was fastened to rigid rod 4 coupled to a vertical coordinate device, which controlled the location of the sensor along the axial coordinate with an error not exceeding ±0.2 mm. The sensor was moved by a microscrew. The moment of contact was detected visually (light-emitting diode 3), and the radial coordinate was recorded by readings of dial indicator 2 with an error not greater than ±0.01 mm. The thickness of the liquid layer was determined as the difference in the coordinates of contacts with the film surface and the wall.

The position of the "dry" wall at different rotational speeds was measured previously to take into account possible vibrations of the cylinder in rotation. The data in the form of approximating relations were used as corrections in the processing of experimental results.

According to the analysis of the isothermal problem in [3, 4], the film thickness in this process is a function of several variables:

$$\delta = f(r_0, H, G_m, \mu, g, \omega, \rho).$$

Here r_0 and H are the inside radius and height of the cylinder, G_m is the mass-flow rate of the liquid, μ and ρ are the dynamic viscosity and density of the liquid, g is the acceleration of gravity, and ω is the rotational speed of the device.

Following dimension theory, we write the following dimensionless complexes:

$$\delta/r_0, \quad H/r_0, \quad \mathrm{Re} = 4\Gamma/\nu, \quad \mathrm{Re}_\omega = \omega r_0{}^2/\nu, \quad B = \omega^2 r_0/g. \tag{1}$$

Here δ/r_0 is the dimensionless thickness of the film, H/r_0 is the relative height of the cylinder, Re is the Reynolds film number derived from the hydraulic diameter of the film cross section, Re_ω is the Reynolds rotational number, B is the overload number, and $\Gamma = G_m/(2\pi r_0 \rho)$ is the spraying density.

For adequate physical simulation and planning of experimental studies, it is necessary that the ranges of numerical values of the dimensionless complexes (1), which are characteristic of a particular technological process, be equal to the ranges of these parameters used in determining the boundaries of the region of experimental space.

We determine the range of dimensional technological parameters for molten glass. According to the data of [9], the capacity of commercial sprayers used on mineral-wool production is 2000–3000 kg/h. The density of glass varies from 1500 to 2500 kg/m^3, depending on the elemental composition. The radius of the sprayer is equal to the radius of the homogenizer and is determined by the possibility of placing a plasma-generator of required power in the interior cavity of the sprayer. According to tentative data obtained for the experimental setup [8], the radius of the cavity is 0.1–0.15 m. In the manufacture of mineral wool, the length and thickness of the fibers produced are largely determined by the viscosity of the melts and the rotational speed of the sprayer. The viscosity is ensured within the range 0.01–0.7 Pa·sec, and the rotational speed [10] is set equal to 150–500 sec^{-1}, depending on the type of fiber (from ultrathin to reinforcing fibers).

From the aforesaid, for dimensionless complexes (1) we should write

$$2 < \mathrm{Re} < 800, \qquad 10^4 < \mathrm{Re}_\omega < 10^7, \qquad 200 < B < 3000. \tag{2}$$

For each cylinder, we performed a two-factor Latin-square experiment in a definite section across its height. The varied factors were the flow rate and the angular frequency of rotation of the cylinder, from which the dimensionless complexes (1) were constructed using geometrical and physical characteristics of the process. In each unit of the experimental arrangement, from three to five measurements were conducted, depending on the reproducibility of results.

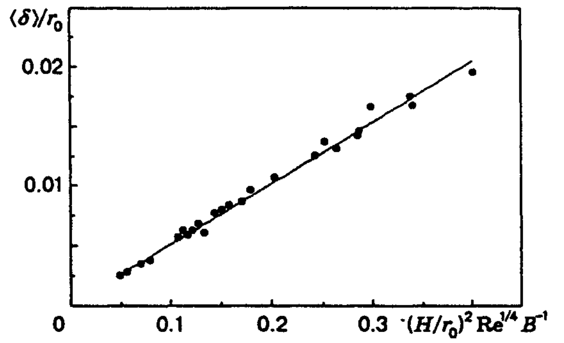

Fig. 2

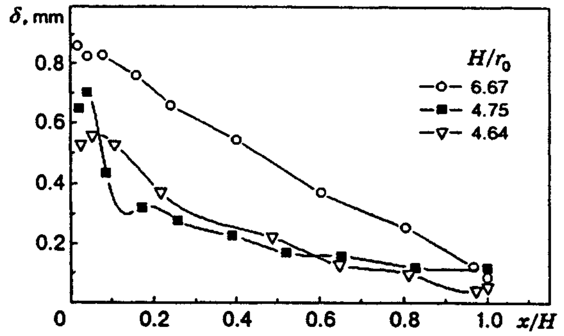

Fig. 3

Fig. 2. Linearized dependence of the liquid-film thickness averaged over the cylinder length (points refer to the experiment).

Fig. 3. Characteristic free-surface profiles of the liquid moving over the inner surface of the rotating cylinder.

In all experiments, the dimensional parameters varied within the following limits:

$$0.72 < G_m < 56.63 \text{ g/sec}, \qquad 74 < \omega < 460 \text{ sec}^{-1}. \tag{3}$$

According to (3), the dimensionless parameters (2) varied within the limits

$$12 < \mathrm{Re} < 700, \qquad 112{,}000 < \mathrm{Re}_\omega < 640{,}000, \qquad 22 < B < 865. \tag{4}$$

According to [11], the values of (4) suggest complete self-similarity of the process for the Reynolds rotational number Re_ω. The relation for the dimensionless film thickness in each section across the height of the cylinder should be sought as the two-parameter function

$$\delta/r_0 = c\,\mathrm{Re}^m B^n, \tag{5}$$

where c, m, and n are approximating coefficients.

Using the Brandon method [12], for each axial coordinate of the cylinder studied we constructed approximating relations in the form (5), from which we determined free-surface profiles of the moving liquid layer for selected values of the numbers Re and B for each cylinder used.

The profiles obtained were used to calculate the integral average thickness of the film $\langle\delta\rangle$ from the heights of the cylinders. Approximation of the values obtained with a general correlation coefficient of 0.96 and prediction error not higher than 10% gives the relation

$$\langle\delta\rangle/r_0 = 0.05\,(H/r_0)^2\,\mathrm{Re}^{1/4}B^{-1}. \tag{6}$$

The approximation of experimental data by relation (6) is illustrated in Fig. 2.

All free-surface profiles of the liquid (Fig. 3) have a distinct maximum in the lower part of the cylinder. The experiment was conducted at Re = 100 and $B = 500$. The maximum is explained by the initial speed distribution in the film and by the fact that, while rotating at the bottom of the device, the flow overcomes friction forces and gravity on the vertical wall. With increase in the radius of the cylinder and the flow rate, the height of the thickening increases and its extent decreases. This is explained by the fact that the liquid supplied to the central part of the disk that forms the cylinder bottom is more heavily accelerated to the place of flow rotation. "Recoil" from the vertical surface occurs. As a result, when the acceleration segment is longer in the formation of the profile on the inlet segment, the radial speed component, directed to the center of the cylinder, has a higher value.

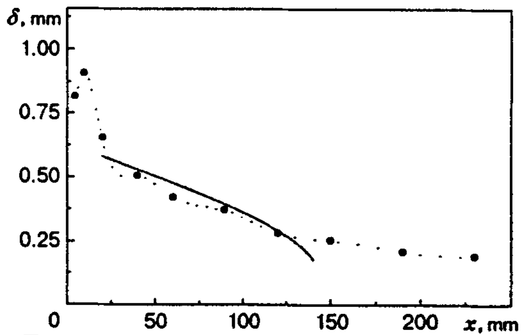

Fig. 4. Free-surface profile of the liquid film: the points refer to the experiment and the solid curve is an analytical solution for laminar flow [4].

In the region of the maximum layer thickness, a hydraulic jump occurs which is similar to the one that arises when the liquid flows along the horizontal surface under conditions of spreading of the incident jet [13]. Behind the leading edge of the thickening there is a toroidal vortex, whose presence in the region of a hydraulic jump on a horizontal plate was detected in the experiment of [14]. Indirect evidence for the existence of return flow in the lower part of the liquid film moving on the inner surface of the rotating cylinder is the presence of thickening of the liquid layer in its nonflow rotary motion. The thickening in the free surface profile is preserved for a rather long time after termination of supply of the model liquid. This is explained by gradual decay of the vortex due to energy dissipation in viscous friction [15]. However, this time is somewhat longer than the time of relaxation of the vortex τ characteristic for the scales of thickening measured in the experiment. The latter is estimated as $\tau = L/(\Omega\nu)^{1/2}$, where L is the characteristic dimension (in our case, half the depth in the region of the jump), Ω is the initial rotational speed of the vortex, proportional to the flow-rate averaged speed of the liquid, and ν is the liquid viscosity. This can be explained by the lag of internal liquid layers, for which the time required to attain "rigid-body" rotation is longer.

Behind the maximum, the film is curved, and this is explained by gradual relaxation of disturbances and the tendency of the flow to enter the regime characterized by a linear free-surface profile under conditions of hydrostatic equilibrium. For short and broad cylinders (two lower curves in Fig. 3), there is a sharp decrease in the layer depth behind the narrow region of the hydraulic jump, after the film thickness changes insignificantly. The viscous-friction forces tend to suppress the flow perturbations produced by the inertia and geometry of the flow. In this connection, the height of the jump and, hence, the mean thickness of the film will apparently decrease with increase in the liquid viscosity. This can explain the fact that the mean thickness of the moving film is described by relation (6), where the viscosity is included in the denominator.

Comparison of the experimental profiles (Re = 206, $B = 433$, and $H/r_0 = 4.64$) with the analytical solution for a laminar film obtained in [4] gives satisfactory agreement for the central (on the axial coordinate) region of the flow. It is evident that the calculated curve is close to the experimental points only for a certain region of the cylinder (Fig. 4). Analysis shows that downstream the friction law changes and the laminar approximation, which is based on the parabolic distribution of the axial speed of the film on the radial coordinate, cannot be realized. The formation of the film surface profile is affected by both the characteristics of the inlet segment and flow conditions.

CONCLUSIONS

1. A series of experiments is performed and free-surface profiles for a liquid layer moving in a mass-force field on the inner surface of a rotating vertical cylinder are constructed.

2. An empirical dependence of the mean film thickness is obtained for the initial segment of the fluid flow under conditions of free flow over the sharp edge of the rotated cylinder.

3. In the lower part of the cylinder there is a hydraulic jump, which strongly affects the flow structure and whose axial position depends weakly on the change in the flow rate and the angular frequency of rotation of the device.

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